Technical Comments

Comment on "Rocket Motor with Electric Acceleration in the Throat"

H. A. HASSAN*
North Carolina State University, Raleigh, N. C.

IN a recent note, Rosciszewski[1] has stated: "At the critical section $M_1 = 1, p = 1, u = 0, du/d\xi = \infty$, and $dp/d\xi = \infty$; however, dp/du and $du^2/d\xi$ are finite." (Bars have been dropped for the sake of convenience.) The statement regarding $du/d\xi$ and $dp/d\xi$ is incorrect since such a statement may result in an infinite rise in velocity and pressure immediately downstream of the channel entrance. It will also be shown that $du^2/d\xi = 0$ at $\xi = 0$.

Since the pressure and velocity are finite throughout, the vanishing of the denominator must be accompanied by the vanishing of the numerator,[2] i.e., at the critical point,

$$\Lambda = 1 \qquad \text{or} \qquad \Lambda = \gamma/(\gamma - 1) \tag{1}$$

with the second condition holding for continuous acceleration from subsonic to supersonic speeds.

With Λ known as $\gamma/(\gamma - 1)$, $du/d\xi$ and $dp/d\xi$ are both of the form 0/0. L'Hospital's rule, applied to Eqs. (10) and (11), yields, at $\xi = 0$,

$$\frac{du}{d\xi} = \frac{1}{\gamma(\gamma - 1)} \frac{(\gamma - 1)(d\Lambda/dp) - \gamma(du/dp)}{1 - (du/dp)} \tag{2}$$

and

$$\frac{dp}{d\xi} = \frac{1}{(\gamma - 1)} \frac{1 - (\gamma - 1)[(d\Lambda/dp) - (du/dp)]}{1 - (du/dp)} \tag{3}$$

where

$$\frac{du}{dp} = -\frac{1}{2}\left\{\frac{2}{\gamma - 1} - \frac{d\Lambda}{dp} \pm \left[\left(\frac{2}{\gamma - 1} - \frac{d\Lambda}{dp}\right)^2 + \frac{4}{\gamma}\frac{d\Lambda}{dp}\right]^{1/2}\right\} \tag{4}$$

For accelerating flows $du/dp < 0$, hence if $d\Lambda/dp \geq 0$, the + sign is appropriate. On the other hand, if $d\Lambda/dp < 0$, both signs yield $du/dp < 0$. Because $du/d\xi$ is finite at $\xi = 0$, $du^2/d\xi = 0$ at $\xi = 0$.

Since $d\Lambda/d\xi$ was specified in Ref. 1, $d\Lambda/dp$ can be calculated from $d\Lambda/d\xi$ by employing Eqs. (3), (4), and the relation

$$d\Lambda/d\xi = (d\Lambda/dp)dp/d\xi \tag{5}$$

The starting values and the starting derivatives give the information needed to start the numerical integration. The integration when Λ is constant can, however, be carried out analytically.[3]

References

[1] Rosciszewski, J., "Rocket motor with electric acceleration in the throat," J. Spacecraft Rockets **2**, 278–280 (1965).
[2] Resler, E. L., Jr. and Sears, W. R., "The prospects for magneto-aerodynamics," J. Aeronaut. Sci. **25**, 235–245, 258 (1958).
[3] Oates, G. C., "Constant-electric-field and constant-magnetic-field magnetogasdynamic channel flow," J. Aerospace Sci. **29**, 231–232 (1962).

Received May 10, 1965.
* Professor of Mechanical Engineering. Member AIAA.

Reply by Author to H. A. Hassan

JAN ROSCISZEWSKI*
General Dynamics/Convair, San Diego, Calif.

THE author does not agree with Hassan's statement that infinite values of $du/d\xi_{\xi=0}$ and $dp/d\xi_{\xi=0}$ must necessarily lead to $u = \infty, p = \infty$ downstream.

As an example, take $du/d\xi \sim \xi^{-1/2}$. Therefore, $du/d\xi_{\xi=0} = \infty$, but integrating ($u = 0$ at $\xi = 0$) one gets $u \sim \xi^{1/2}$, and u is finite for $\xi \geqslant 0$ (similar considerations are valid for p). In fact this is the type of singularity in my problem.

In my paper, gas is accelerated electrically starting from the sonic conditions and there is no need for having $\Lambda = 1$ or $\Lambda = \gamma/(\gamma - 1)$, which would be required for pure electrical acceleration from subsonic to supersonic speeds in a constant cross-sectional channel.

For $\Lambda \neq 1$ and $\Lambda \neq \gamma/(\gamma - 1)$, the quantity $du^2/d\xi_{\xi=0}$ is finite. Therefore, the starting procedure of my paper is correct, and, in addition, smooth integral curves were obtained near the singular point.

The difference of opinion comes from the fact that Hassan based his remarks on the references where the pure electric acceleration from subsonic to supersonic flow was considered. This is not the case in my paper.

Received May 28, 1965; revision received June 14, 1965.
* Consultant, Space Science Laboratory. Member AIAA.

Comment on "Spinning Vehicle Nutation Damper"

E. J. SLACHMUYLDERS*
European Space Technology Centre, Delft, the Netherlands

RECENTLY, Wadleigh, Galloway, and Mathur[1] have described a nutation damper consisting of a mass sliding along a rectilinear guide. Starting from the dynamic equations of the system, curves are obtained showing optimum design parameters for this particular nutation damper.

It should be noted, however, that the equations used as a starting point are not complete. Indeed, when evaluating the torque exerted on the satellite by the sliding mass, it is necessary to consider the total acceleration of this mass instead of its relative acceleration along the guide. This yields the following equations:

$$\dot{p}(I_x + my^2) - may\dot{q} = -m(2y\dot{y}p + aypr + qry^2) \tag{1}$$

$$\dot{q}(I_y + ma^2) - may\dot{p} = pr(I_z - I_x) + m(2ap\dot{y} + pra^2 + ayqr) \tag{2}$$

$$\dot{r}(I_z + my^2) = pq(I_x - I_y) + m\{pqy^2 - 2y\dot{y}r + ay(\omega_n^2 - q^2 - r^2) + 2\lambda\omega_n a\dot{y}\} \tag{3}$$

$$\ddot{y} + 2\lambda\omega_n\dot{y} - (r^2 + p^2 - \omega_n^2)y + apq + a\dot{r} = 0 \tag{4}$$

Received March 29, 1965.
* Engineer, Applied Research Department.

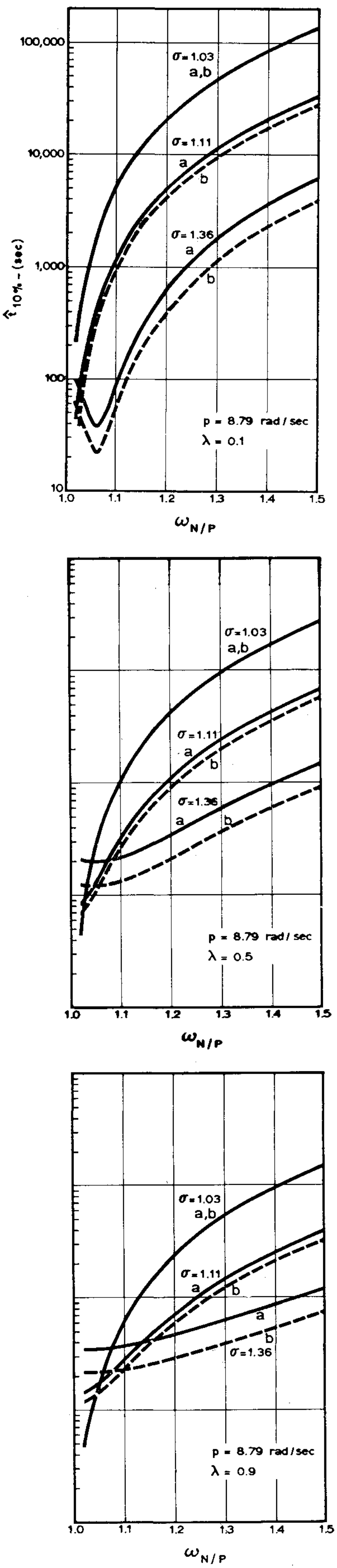

Fig. 1 Computer solution for damping times: $ma^2/Iy = 6.46 \times 10^{-4}$ $p = 8.79$ rad/sec; a) previous results and b) new results.

It can be shown that this set of equations satisfies the principle of conservation of angular momentum of the system consisting of the satellite body and the sliding mass, i.e.,

$$\{p(I_x + my^2) - mayq\}^2 + \{q(I_y + ma^2) - mayp\}^2 + \{r(I_z + ma^2 + my^2) + ma\dot{y}\}^2 = \text{const}$$

The present set of equations is far more complex than the one treated in the reference. In order to get an estimate of the importance of the differences between the two sets of equations, the present set is linearized and the resulting characteristic equation is solved to yield $t_{10\%}$ (the time required to damp to 10%). The results for the case, where $p = 8.79$ rad/sec, are shown in Fig. 1. The solid lines represent the values taken from the reference, and the dashed lines represent the values obtained from the present equations.

Although differences of up to 40% occur, it is seen that the general shape of the curves is not affected. In particular, the location of the optimum has not changed. It is interesting to note that the difference increases with increasing ratio of moments of inertia, but is independent of the damping coefficient.

No attempt has been made to evaluate the effect of the nonlinear terms in the complete equations.

Reference

[1] Wadleigh, K. H., Galloway, A. J., and Mathur, P. N., "Spinning vehicle nutation damper," J. Spacecraft Rockets **1**, 588–592 (1964).

Reply by Authors to E. J. Slachmuylders

K. H. Wadleigh* and A. J. Galloway*
The Bendix Corporation, Ann Arbor, Mich.

The foregoing Technical Comment discusses a more rigorous and complete set of equations of motion than were used in arriving at the computed curves of Ref. (1). This was pointed out also in private correspondence by P. C. Wheeler of Stanford Electronics Laboratories, Stanford, Calif. Linearization of the complete equations results in the same set of equations of motion with the exception of an added term in the second equation of the set at the bottom of p. 589 of Ref. (1) as follows (the added term is enclosed in square brackets):

$$\dot{q} + \Omega r - [(2\,\text{map}/I_y)\dot{y}] = 0$$

When this term is included in the determinant expansion to obtain the characteristic function, a term is added to the coefficient of s^2 leaving the coefficients of s^3 and s (which contain the damping factor λ) unaffected.

A calculation, including the enclosed term, gave a time to damp some 30% less than the curve value with $p = 18.8$ rad/sec, $\lambda = 0.5$, $\sigma = 1.36$, and $\omega_n/p = 1.10$.

Likely, the error caused by omission of the term is of the order of magnitude of other errors in the system caused by linearization. However, the authors are indebted to E. J. Slachmuylders and P. C. Wheeler for pointing out the complete equations.

References

[1] Wadleigh, K. H., Galloway, A. J., and Mathur, P. N., "Spinning vehicle nutation damper," J. Spacecraft Rockets **1**, 588–592 (1964).

Received April 2, 1965.
* Bendix Systems Division.